III Semester B.A./B.Sc. Examination, November/December 2018
(Semester Scheme) (CBCS) (F + R) (2015-16 and Onwards)
MATHEMATICS – III
Mathematics

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) Find the number of generators of the cyclic group of order 60.

 b) Find all the left cosets of $H = \{0, 4, 8\}$ in $(Z_{12}, +_{12})$.

 c) Test the nature of the sequence $\{n[\log (n + 1) - \log n]\}$.

 d) Examine the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$.

 e) Test the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$

 f) Find the value of 'C' using Rolle's theorem for the function $f(x) = 8x - x^2$ in $[2, 6]$.

 g) State Lagrange's mean value theorem.

 h) Evaluate $\lim_{x \to 0} (\operatorname{cosec} x - \cot x)$.

PART – B

Answer **one full** question : **(1×15=15)**

2. a) In a group G, prove that $O(a) = O(a^{-1})$ $\forall a \in G$.

 b) Find the number of generators of the cyclic group of order 8. If 'a' is one of the generator, then what are the other generators ?

 c) State and prove Euler's theorem.

OR

3. a) Any two right (left) cosets of a subgroup H of a group G are either disjoint or identical.

 b) Define cyclic group. Show that every cyclic group is abelian.

 c) If G is a finite group and H is a subgroup of G, then the order of H divides the order of G.

P.T.O.

PART – C

Answer **two full** questions : (2×15=30)

4. a) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty} (a_n + b_n) = a + b$.

   b) Prove that a monotonically increasing sequence which is bounded above is convergent.

   c) Examine the convergence of the sequence

      i) $\left\{\left(1+\frac{2}{n}\right)^n\right\}$

      ii) $\left\{\sqrt{n+1}-\sqrt{n}\right\}$

OR

5. a) Prove that the sequence $\left\{\frac{3n+4}{2n+1}\right\}$ is

      i) Monotonically decreasing

      ii) Bounded

      iii) Converges to $\frac{3}{2}$.

   b) Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{n+n}$ is convergent.

   c) Find limit of the sequence 0.4, 0.44, 0.444, ...

6. a) State and prove D'Alembert's ratio test for the series of positive terms.

   b) Examine the convergence of the series $\frac{x^2}{2\sqrt{1}} + \frac{x^3}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \ldots\ldots$

   c) Sum the series to infinity $\frac{1}{6} + \frac{1.4}{6.12} + \frac{1.4.7}{6.12.18} + \ldots\ldots$

OR

7. a) State and prove Leibnitz test on Alternating Series.

   b) Examine the convergence of the series $\sum\left(\frac{nx}{n+1}\right)^n$.

   c) Sum to infinity of the series $\frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \frac{5^2}{4!} + \ldots\ldots$

PART – D

Answer **one full** question : **(1×15=15)**

8. a) Discuss the continuity of

$$f(x) = \begin{cases} 1+x & \text{for } x<2 \\ 5-x & \text{for } x \geq 2 \end{cases} \text{ at } x = 2.$$

b) State and prove Rolle's theorem.

c) Evaluate :

i) $\lim_{x \to 0} \left(\frac{a^x - b^x}{x} \right)$

ii) $\lim_{x \to 0} (1 + \sin x)^{\cot x}$

OR

9. a) Examine the differentiability of the function f(x) defined by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1-x & \text{for } x<1 \end{cases} \text{ at } x = 1.$$

b) State and prove Cauchy's mean value theorem.

c) Expand tan x up to the term containing x^3 by using Maclaurin's expansion.